

AP Statistics

Solutions to Packet 9

Sampling Distributions

Sampling Distributions

Sample Proportions

Sample Means

HW #7 1 – 4, 9, 10 – 11

For each boldface number in Exercises 9.1 – 9.4, (a) state whether it is a parameter or a statistic and (b) use appropriate notation to describe each number; for example, $p = 0.65$.

9.1 MAKING BALL BEARINGS A carload lot of ball bearings has mean diameter **2.5003** centimeters (cm). This is within the specifications for acceptance of the lot by the purchaser. By chance, an inspector chooses 100 bearings from the lot that have mean diameter **2.5009** cm. Because this is outside the specified limits, the lot is mistakenly rejected.

$m = 2.5003$ is a parameter; $\bar{x} = 2.5009$ is a statistic.

9.2 UNEMPLOYMENT The Bureau of Labor Statistics last month interviewed 60,000 members of the U.S. labor force, of whom **7.2%** were unemployed.

$\hat{p} = 7.2\%$ is a statistic.

9.3 TELEMARKETING A telemarketing firm in Los Angeles uses a device that dials residential telephone numbers in that city at random. Of the first 100 numbers dialed, **48%** are unlisted. This is not surprising because **52%** of all Los Angeles residential phones are unlisted.

$\hat{p} = 48\%$ is a statistic, $p = 52\%$ is a parameter.

9.4 WELL-FED RATS A researcher carries out a randomized comparative experiment with young rats to investigate the effects of a toxic compound in food. She feeds the control group a normal diet. The experimental group receives a diet with 2500 parts per million of the toxic material. After 8 weeks, the mean weight gain is **355** grams for the control group and **289** grams for the experimental group.

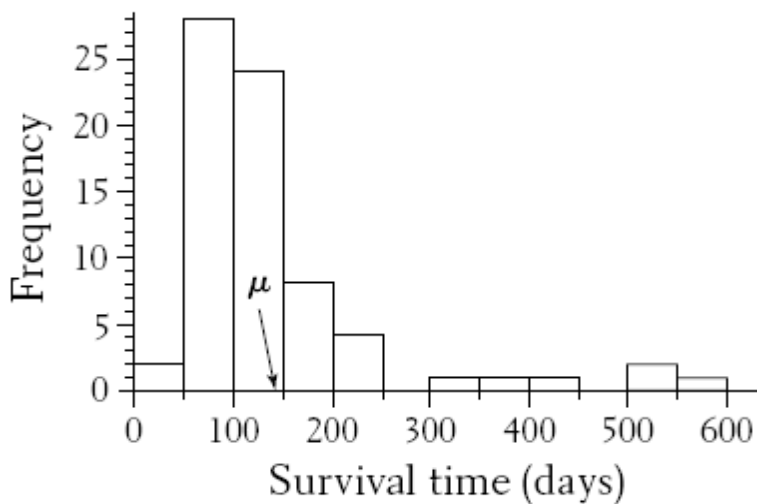
Both $\bar{x}_1 = 355$ and $\bar{x}_2 = 289$ are statistics.

9.9 **GUINEA PIGS** Link the data of survival time for guinea pigs from another calculator. Consider these 72 animals to be the population of interest.

Survival time (days) of guinea pigs in a medical experiment.

43	45	53	56	56	57	58	66	67	73	74	79
80	80	81	81	81	82	83	83	84	88	89	91
91	92	92	97	99	99	100	100	101	102	102	102
103	104	107	108	109	113	114	118	121	123	126	128
137	138	139	144	145	147	156	162	174	178	179	184
191	198	211	214	243	249	329	380	403	511	522	598

(a) Make a histogram of the 72 survival times. This is the population distribution. It is strongly skewed to the right.



(b) Find the mean of the 72 survival times. This is the population mean μ . Mark μ on the x-axis of your histogram. **For the 72 survival times, $\mu = 141.847$ days.**

(c) Label the members of the population 01 to 72 and use Table B to choose a SRS of size $n = 12$. List your sample and find the mean \bar{x} of your sample. **All answers vary.**

_____ $\bar{x} =$ _____

Mark the value of \bar{x} with a point on the axis of your histogram.

(d) Repeat this three times.

_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	$\bar{x} =$ _____
_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	$\bar{x} =$ _____
_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	_____	$\bar{x} =$ _____

Mark these values on the axis of your histogram.

Would you be surprised if all four values of \bar{x} lie on the same side of μ ? _____

It would be unlikely (though not impossible) for all four \bar{x} values to fall on the same side of μ . This is one implication of the unbiasedness of \bar{x} : Some values will be higher and some lower than μ . (But note, it is not necessarily half and half.)

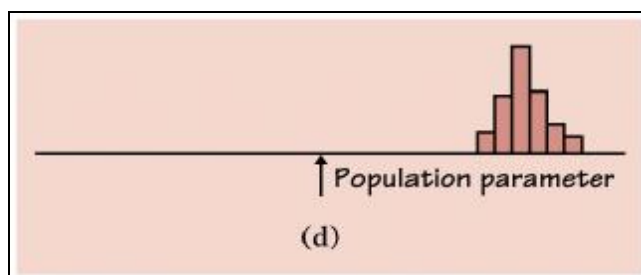
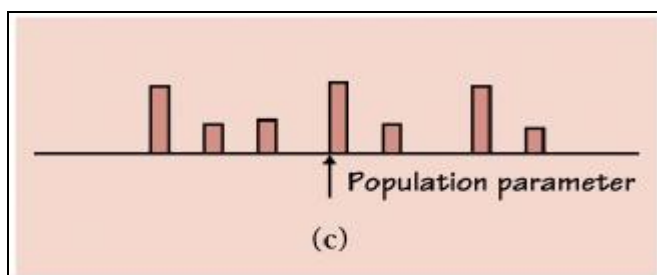
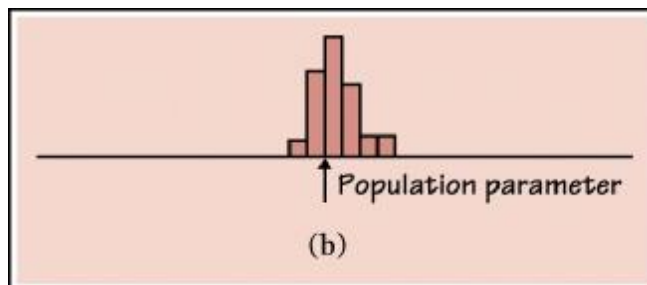
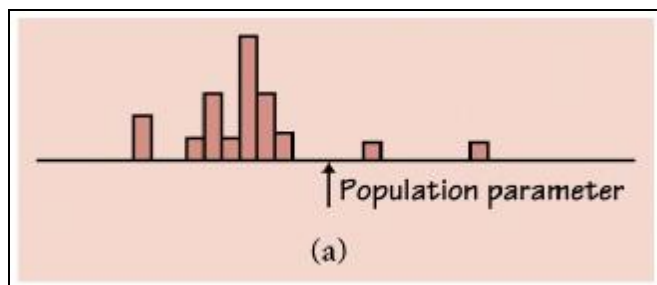
(e) If you chose all possible SRSs of size 12 from this population and made a histogram of the \bar{x} -values, where would you expect the center of this sampling distribution to lie? _____

The mean of the (theoretical) sampling distribution would be μ .

(f) Pool your results with those of your classmates to construct a histogram of the \bar{x} values you obtained. Describe the shape, center, and spread of this distribution. Is the histogram approximately normal?

Discuss in class.

9.10 **BIAS AND VARIABILITY** The figure below shows histograms of four sampling distributions of statistics intended to estimate the same parameter. Label each distribution relative to the others as having large or small bias and as having large or small variability.



- (a) Large bias and large variability.
 (c) Small bias, large variability.

- (b) Small bias and small variability.
 (d) Large bias, small variability.

9.11 **IRS AUDITS** The Internal Revenue Service plans to examine an SRS of individual federal income tax returns from each state. One variable of interest is the proportion of returns claiming itemized deductions. The total number of tax returns in each state varies from almost 14 million in California to fewer than 210,000 in Wyoming.

(a) Will the sampling variability of the sample proportion change from state to state if an SRS of 2000 tax returns is selected in each state? Explain your answer.

Since the smallest number of total tax returns (i.e., the smallest population) is still more than 100 times the sample size, the variability will be (approximately) the same for all states.

(b) Will the sampling variability of the sample proportion change from state to state if an SRS of 1% of all tax returns is selected in each state? Explain your answer.

Yes, it will change—the sample taken from Wyoming will be about the same size, but the sample in, e.g., California will be considerably larger, and therefore the variability will decrease.

9.17 **SCHOOL VOUCHERS** A national opinion poll recently estimated that 44% ($\hat{p} = 0.44$) of all adults agree that parents of school-age children should be given vouchers good for education at any public or private school of their choice. The polling organization used a probability sampling method for which the sample proportion \hat{p} has a normal distribution with standard deviation about 0.015. If a sample were drawn by the same method from the state of New Jersey (population 7.8 million) instead of from the entire United States (population 280 million), would this standard deviation be larger, about the same, or smaller? Explain your answer.

The variability would be practically the same for either population.

9.19 **DO YOU DRINK THE CEREAL MILK?** A USA *Today* poll asked a random sample of 1012 U.S. adults what they do with the milk in the bowl after they have eaten the cereal. Of the respondents, 67% said that they drink it. Suppose that 70% of U.S. adults actually drink the cereal milk.

(a) Find the mean and standard deviation of the proportion \hat{p} of the sample that say they drink the cereal

milk. $m = p = 0.7; s = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.7)(0.3)}{1012}} = 0.0144$

(b) Explain why you can use the formula for the standard deviation of \hat{p} in this setting (rule of thumb 1)

The population (all U.S. adults) is clearly at least 10 times as large as the sample (the 1012 surveyed adults).

(c) Check that you can use the normal approximation for the distribution of \hat{p} (rule of thumb 2).

$$np = 1012(0.7) = 708.4 \geq 10$$

$$n(1-p) = 1012(0.3) = 303.6 \geq 10$$

(d) Find the probability of obtaining a sample of 1012 adults in which 67% or fewer say they drink the cereal milk. Do you have any doubts about the result of this poll?

$$P(\hat{p} \leq 0.67) = P\left(Z \leq \frac{0.67 - 0.7}{0.0144}\right) = P(Z \leq -2.0833) = 0.0186, \text{ this is an unusual result if 70\% of}$$

the population actually drinks the cereal milk. This would occur less than 2% of the time by chance alone.

(e) What sample size would be required to reduce the standard deviation of the sample proportion to one-half the value you found in (a)?

To reduce the std dev by $\frac{1}{2}$ we would have to multiply the sample size by 4; we would need to sample $(1012)(4) = 4048$ adults.

(f) If the pollsters had surveyed 1012 teenagers instead of 1012 adults, do you think the sample proportion \hat{p} would have been greater than, equal to, or less than 0.67? Explain.

It would probably be higher, since teenagers (and children in general) have a greater tendency to drink the cereal milk.

9.20 **DO YOU GO TO CHURCH?** The Gallup Poll asked a probability sample of 1785 adults whether they attended church or synagogue during the past week. Suppose that 40% of the adult population did attend. We would like to know the probability that an SRS of size 1785 would come within plus or minus 3 percentage points of this true value.

(a) If \hat{p} is the proportion of the sample who did attend church or synagogue, what is the mean of the sampling distribution of \hat{p} ? What is its standard deviation?

$$m = p = 0.4; \quad s = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.4)(0.6)}{1785}} = 0.0116$$

(b) Explain why you can use the formula for the standard deviation of \hat{p} in this setting (rule of thumb 1).
The population (all U.S. adults) is clearly at least 10 times as large as the sample (the 1785 surveyed adults).

(c) Check that you can use the normal approximation for the distribution of \hat{p} (rule of thumb 2).

$$np = 1785(0.4) = 714 \geq 10$$

$$n(1-p) = 1785(0.6) = 1071 \geq 10$$

(d) Find the probability that \hat{p} takes a value between 0.37 and 0.43. Will an SRS of size 1785 usually give a result \hat{p} within plus or minus 3 percentage points of the true population proportion? Explain.

$$P(0.37 < \hat{p} < 0.43) = P\left(\frac{0.37-0.4}{0.0116} < Z < \frac{0.43-0.4}{0.0116}\right) = P(-2.586 < Z < 2.586) = 0.994$$

Over 99% of all samples should give \hat{p} within $\pm 3\%$ of the true population proportion.

9.21 **DO YOU GO TO CHURCH?** Suppose that 40% of the adult population attended church or synagogue last week. Exercise 9.20 asks the probability that \hat{p} from an SRS estimates $p = 0.4$ within 3 percentage points. Find this probability for SRSs of sizes 300, 1200, and 4800. What general fact do your results illustrate?

$$\text{For } n = 300: s = 0.02828 \text{ and } P = 0.7108$$

$$\text{For } n = 1200: s = 0.01414 \text{ and } P = 0.9660$$

$$\text{For } n = 4800: s = 0.00707 \text{ and } P \approx 1$$

Larger sample sizes give more accurate results (the sample proportions are more likely to be close to the true proportion).

9.22 **HARLEY MOTORCYCLES** Harley-Davidson motorcycles make up 14% of all the motorcycles registered in the United States. You plan to interview an SRS of 500 motorcycle owners.

(a) What is the approximate distribution of your sample who own Harleys?

The distribution is approximately normal with mean $m = p = 0.14$

and standard deviation $s = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.14)(0.86)}{500}} = 0.0155$

(b) How likely is your sample to contain 20% or more who own Harleys? Do a normal probability calculation to answer this question.

The probability that the sample will contain 20% or more Harley owners is unlikely!

$$P(\hat{p} > 0.20) = P\left(Z > \frac{0.20 - 0.14}{0.0155}\right) = P(Z > 3.87) < 0.0002.$$

(c) How likely is your sample to contain at least 15% who own Harleys? Do a normal probability calculation to answer this question.

There is a fairly good chance of finding at least 15% Harley owners (We would expect to see this result about 26% of the time)

$$P(\hat{p} > 0.15) = P\left(Z > \frac{0.15 - 0.14}{0.0155}\right) = P(Z > 0.64) = 0.2611.$$

9.23 **ON-TIME SHIPPING** Your mail-order company advertises that it ships 90% of its orders within three working days. You select an SRS of 100 of the 5000 orders received in the past week for an audit. The audit reveals that 86 of these orders were shipped on time.

(a) What is the sample proportion of orders shipped on time? $\hat{p} = 0.86$

(b) If the company really ships 90% of its orders on time, what is the probability that the proportion in an SRS of 100 orders is as small as the proportion in your sample or smaller?

We use the normal approximation (Rule of Thumb 2 is just satisfied— $n(1-p) = 10$).

The standard deviation is $\sqrt{\frac{0.9(0.1)}{100}} = 0.03$, and

$$P(\hat{p} \leq 0.86) = P\left(Z \leq \frac{0.86 - 0.9}{0.03}\right) = P(Z \leq -1.33) = 0.0918$$

(Note: The exact probability is 0.1239.)

9.24 Exercise 9.22 asks for probability calculations about Harley-Davidson motorcycle ownership. Exercise 9.23 asks for a similar calculation about a random sample of mail orders. For which calculation does the normal approximation to the sampling distribution of \hat{p} give a more accurate answer? Why? (You need not actually do either calculation.)

The calculation for Exercise 9.22 should be more accurate. This calculation is based on a larger sample size (500, as opposed to the 100 of Exercise 9.23). Rule of Thumb 2 is easily satisfied in Exercise 9.22 but just barely satisfied in Exercise 9.23.

9.30 **RULES OF THUMB** Explain why you cannot use the methods of this section to find the following probabilities.

(a) A factory employs 3000 unionized workers, of whom 30% are Hispanic. The 15-member union executive committee contains 3 Hispanics. What would be the probability of 3 or fewer Hispanics if the executive committee were chosen at random from all the workers?

$np = (15)(0.3) = 4.5$ — this fails Rule of Thumb 2.

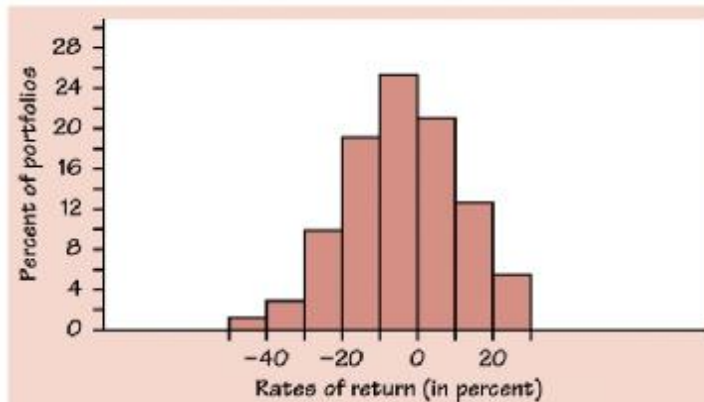
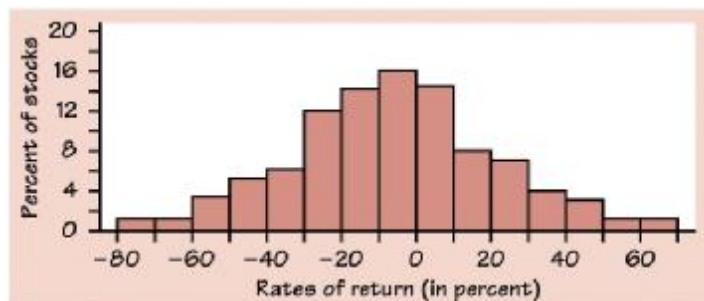
(b) A university is concerned about the academic standing of its intercollegiate athletes. A study committee chooses an SRS of 50 of the 316 athletes to interview in detail. Suppose that in fact 40% of the athletes have been told by coaches to neglect their studies on at least one occasion. What is the probability that at least 15 in the sample are among this group?

The population size (316) is not at least 10 times as large as the sample size (50) — this fails Rule of Thumb 1.

(c) Use what you learned in Chapter 8 to find the probability described in part (a).

$P(X \leq 3) = \text{binomcdf}(15, 0.3, 3) = 0.2969$.

9.31 **BULL MARKET OR BEAR MARKET?** Investors remember 1987 as the year the stocks lost 20% of their value in a single day. For 1987 as a whole, the mean return of all common stocks on the New York Stock Exchange was $\mu = -3.5\%$. (That is, these stocks lost an average of 3.5% of their value in 1987.) The standard deviation of the returns was about $\sigma = 26\%$. The figure directly below shows the distribution of returns. The figure below that is the sampling distribution of the mean returns \bar{x} for all possible samples of 5 stocks.



(a) What are the mean and standard deviation of the distribution in the second figure?

$$m = -3.5\%; \quad s = \frac{26\%}{\sqrt{5}} = 11.628\%$$

(b) Assuming that the population distribution of returns on individual common stocks is normal, what is the probability that a randomly chosen stock showed a return of at least 5% in 1987?

$$P(X \geq 5\%) = P\left(\frac{5 - (-3.5)}{26}\right) = P(Z \geq 0.3269) = 0.3719$$

The probability that a randomly chosen stock showed a return of at least 5% is about 37%.

(c) Assuming that the population distribution of returns on individual common stocks is normal, what is the probability that a randomly chosen portfolio of 5 stocks showed a return of at least 5% in 1987?

$$P(\bar{x} \geq 5\%) = P\left(Z \geq \frac{5 - (-3.5)}{11.628}\right) = P(Z \geq 0.73) = 0.2327$$

The probability that a randomly chosen portfolio of 5 stocks showed a return of at least 5% is about 23%

(d) What percentage of 5-stock portfolios lost money in 1987?

$$P(\bar{x} < 0) = P\left(Z < \frac{0 - (-3.5)}{11.628}\right) = P(Z < 0.30) = 0.6179.$$

Approximately 62% of all five-stock portfolios lost money.

9.32 ACT SCORES The scores of individual students on the American College Testing (ACT) composite college entrance examination have a normal distribution with mean 18.6 and standard deviation 5.9.

(a) What is the probability that a single student randomly chosen from all those taking the test scores 21 or higher? $P(X \geq 21) = P(Z \geq 0.4068) = 0.3421$

The probability that a single student randomly chosen from all those taking the test scores 21 or higher is about 34%.

(b) Now take an SRS of 50 students who took the test. What are the mean and standard deviation of the average (sample mean) score for the 50 students? Do your results depend on the fact that individual scores have a normal distribution?

$$m = 18.6; \quad s = \frac{5.9}{\sqrt{50}} = 0.8344. \quad \text{This result is independent of distribution shape.}$$

(c) What is the probability that the mean score \bar{x} of these students is 21 or higher?

$$P(\bar{x} \geq 21) = P\left(Z \geq \frac{21 - 18.6}{0.8344}\right) = P(Z \geq 2.8764) = 0.0020$$

The probability that the mean score of those students is 21 or higher is about 0.2%.

9.34 MEASURING BLOOD CHOLESTEROL A study of the health of teenagers plans to measure the blood cholesterol level of an SRS of youth of ages 13 to 16 years. The researchers will report the mean \bar{x} from their sample as an estimate of the mean cholesterol level μ in this population.

(a) Explain to someone who knows no statistics what it means to say that \bar{x} is an “unbiased” estimator of μ .

If we choose many samples, the average of the \bar{x} -values from these samples will be close to μ . (i.e., \bar{x} is “correct on the average” in many samples.)

(b) The sample result \bar{x} is an unbiased estimator of the population parameter μ no matter what size SRS the study chooses. Explain to someone who knows no statistics why a large sample gives more trustworthy results than a small sample.

The larger sample will give more information, and therefore more precise results; that is, \bar{x} is more likely to be close to the population truth. Also, \bar{x} for a larger sample is less affected by outliers.

9.35 BAD CARPET The number of flaws per square yard in a type of carpet material varies with mean 1.6 flaws per square yard and standard deviation 1.2 flaws per square yard. The population distribution cannot be normal, because a count takes only whole-number values. An inspector studies 200 square yards of the material, records the number of flaws found in each square yard, and calculates \bar{x} , the mean number of flaws per square yard inspected. Use the central limit theorem to find the approximate probability that the mean number of flaws exceeds 2 per square yard.

\bar{x} has approximately a $N(1.6, 0.0849)$ distribution.

The probability the mean number of flaws exceed 2 per square yard is

$$P(X > 2) = P\left(Z > \frac{2 - 1.6}{0.0849}\right) = P(Z > 4.71) - \text{essentially } 0!$$

9.37 COAL MINER’S DUST A laboratory weighs filters from a coal mine to measure the amount of dust in the mine atmosphere. Repeated measurements of the weight of dust on the same filter vary normally with standard deviation $\sigma = 0.08$ milligrams (mg) because the weighing is not perfectly precise. The dust on a particular filter actually weighs 123 mg. Repeated weighings will then have the normal distribution with mean 123 mg and standard deviation 0.08 mg.

(a) The laboratory reports the mean of 3 weighings. What is the distribution of this mean?

$$N\left(123, \frac{0.08}{\sqrt{3}}\right) = N(123, 0.04619)$$

(b) What is the probability that the laboratory reports a weight of 124 mg or higher for this filter?

$$P(\bar{x} \geq 124) = P\left(Z \geq \frac{124 - 123}{0.04619}\right) = P(Z \geq 21.65) - \text{essentially } 0.$$

9.38 **MAKING AUTO PARTS** An automatic grinding machine in an auto parts plant prepares axles with a target diameter $\mu = 40.125$ millimeters (mm). The machine has some variability, so the standard deviation of the diameters is $\sigma = 0.002$ mm. The machine operator inspects a sample of 4 axles each hour for quality control purposes and records the sample mean diameter.

(a) What will be the mean and standard deviation of the numbers recorded? Do your results depend on whether or not the axle diameters have a normal distribution?

Mean: 40.125, standard deviation $s = \frac{0.002}{\sqrt{4}} = 0.001$; normality is not needed.

(b) Can you find the probability that an SRS of 4 axles has a mean diameter greater than 40.127 mm? If so, do it. If not, explain why not.

No: We cannot find $P(\bar{x} > 40.127)$ based on a sample of size 4. The sample size must be larger to justify use of the central limit theorem if the distribution type is unknown.

9.43 **REPUBLICAN VOTERS** Voter registration records show that **68%** of all voters in Indianapolis are registered as Republicans. To test whether the numbers dialed by a random digit dialing device really are random, you use the device to call 150 randomly chosen residential telephones in Indianapolis. Of the registered voters contacted, **73%** are registered Republicans.

(a) Is each of the boldface numbers a parameter or a statistic? Give the appropriate notation for each.

$p = 68\% = 0.68$ is a parameter; $\hat{p} = 73\% = 0.73$ is a statistic.

(b) What are the mean and the standard deviation of the same proportion of registered Republicans in samples of size 150 from Indianapolis?

$$m = p = 0.68; \quad s = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.68)(0.32)}{150}} = 0.0381$$

(c) Find the probability of obtaining an SRS of size 150 from the population of Indianapolis voters in which 73% or more are registered Republicans. How well is your random digit device working?

$P(\hat{p} \geq 0.73) = P\left(Z \geq \frac{0.73 - 0.68}{0.0381}\right) = P(Z \geq 1.3128) = 0.0946$. There is almost a 10% (one in ten) chance that an observation of \hat{p} greater than or equal to the observed value of 0.73 will be seen.

9.46 **POLLING WOMEN** Suppose that 47% of all adult women think they do not get enough time for themselves. An opinion poll interviews 1025 randomly chosen women and records the sample proportion who feel they don't get enough time for themselves.

(a) Describe the sampling distribution of \hat{p} .

\hat{p} has an approximately normal distribution with

$$m = p = 0.47; \quad s = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.47)(0.53)}{1025}} = 0.0156$$

(b) The truth about the population is $p = 0.47$. In what range will the middle 95% of all sample results fall? The middle 95% of all sample results will fall within 2σ ; $\pm 2(0.0156)$; ± 0.0312 of the mean 0.47, that is, in the interval 0.4388 to 0.5012.

(c) What is the probability that the poll gets a sample in which fewer than 45% say they do not get enough time for themselves?

$$P(\hat{p} < 0.45) = P\left(Z < \frac{0.45 - 0.47}{0.0156}\right) = P(Z < -1.282) = 0.0999$$

9.49 **IQ TESTS** The Wechsler Adult Intelligence Scale (WAIS) is a common "IQ test" for adults. The distribution of WAIS scores for persons over 16 years of age is approximately normal with mean 100 and standard deviation 15.

(a) What is the probability that a randomly chosen individual has a WAIS score of 105 or higher?

$$P(X > 105) = P\left(Z > \frac{105 - 100}{15}\right) = P\left(Z > \frac{1}{3}\right) = 0.36944$$

(b) What are the mean and standard deviation of the sampling distribution of the average WAIS score

\bar{x} for an SRS of 60 people? Mean: 100; standard deviation: $s = \frac{15}{\sqrt{60}} = 1.93694$

(c) What is the probability that the average WAIS of an SRS of 60 people is 105 or higher?

$$P(\bar{x} > 105) = P\left(Z > \frac{105 - 100}{1.93649}\right) = P(Z > 2.5820) = 0.00491$$

(d) Would your answers to any of (a), (b), or (c) be affected if the distribution of WAIS scores in the adult population were distinctly nonnormal?

The answer to (a) could be quite different. We could not use the normal distribution to find the probability. (b) would be the same (it does not depend on normality at all). The answer we gave for (c) would still be fairly reliable because of the central limit theorem.

9.52 **HIGH SCHOOL DROPOUTS** High school dropouts make up 14.1% of all Americans aged 18 to 24. A vocational school that wants to attract dropouts mails an advertising flyer to 25,000 persons between the ages of 18 and 24.

(a) If the mailing list can be considered a random sample of the population, what is the mean number of high school dropouts who will receive the flyer?

$$np = (25000)(0.141) = 3525.$$

(b) What is the probability that at least 3500 dropouts will receive the flyer?

$$s = \sqrt{npq} = \sqrt{25000(0.141)(0.859)} = 55.027$$

$$P(X > 3500) = P\left(Z > \frac{3500 - 3525}{55.027}\right) = P(Z > -0.4543) = 0.675$$

The probability that at least 3500 dropouts will receive the flyer is 67.5%.