AP Statistics
Solutions to Packet 2
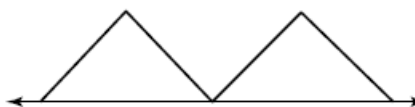
The Normal Distributions
Density Curves and the Normal Distribution
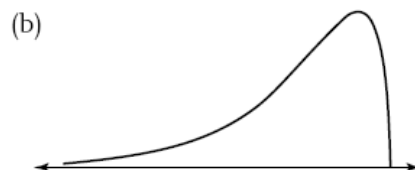Standard Normal Calculations

2.1 DENSITY CURVES

(a) Sketch a density curve that is symmetric but has a shape different from that of the standard normal curve.
   There are many correct answers.

(b) Sketch a density curve that is strongly skewed to the left.

2.2 A UNIFORM DISTRIBUTION

The figure below shows the density curve of a *uniform distribution*. The curve takes the constant value 1 over the interval from 0 to 1 and is zero outside the range of values. This means that data described by this distribution take values that are uniformly spread between 0 and 1. Use areas under this density curve to answer the following questions.
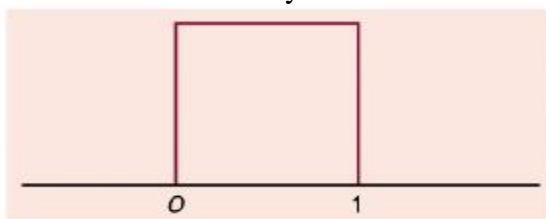
(a) Why is the total area under this curve equal to 1? The area under the curve is a rectangle with height 1 and width 1. Thus the total area under the curve = $1 \times 1 = 1$

(b) What percent of the observations lies above 0.8? 20%. (The region is a rectangle with height 1 and base width 0.2; hence the area is 0.2.)

(c) What percent of the observations lie below 0.6? 60%.

(d) What percent of the observations lie between 0.25 and 0.75? 50%.

(e) What is the mean μ of this distribution? $\mu = 0.5$, the "balance point" of the density curve.

2.4 FINDING MEANS AND MEDIANS

The figure below displays three density curves, each with three points indicated. At which of these points on each curve the mean and median fall?
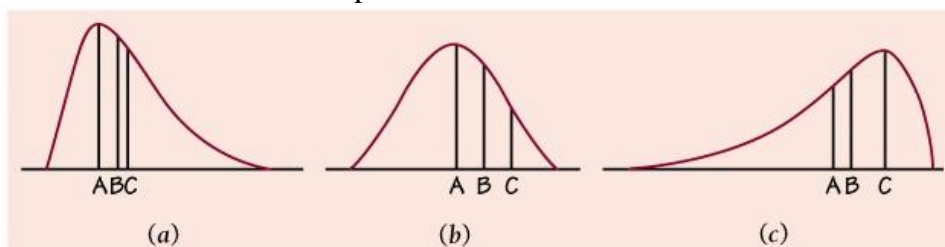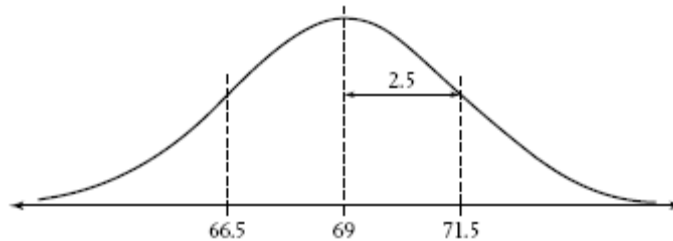
(a) mean __ C __   median __ B ___   (b) mean ___ A ___   median ___ A ___

(c) mean __ A __   median __ B ___

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2.6 **MEN'S HEIGHTS** The distribution of heights of adult American men is approximately normal with mean 69 inches and standard deviation 2.5 inches. Draw a normal curve on which this mean and standard deviation are correctly located. (*Hint: Draw the curve first, locate the points where the curvature changes, than mark the horizontal axis.*)

2.7 **MORE ON MEN'S HEIGHTS**   The distribution of heights of adult American men is approximately normal with mean 69 inches and standard deviation 2.5 inches. Use the 68–95–99.7 rule to answer the following questions.

(a) What percent of men are taller than 74 inches?
       2.5% (this is 2 standard deviations above the mean).

(b) Between what heights do the middle 95% of men fall? 69 ± 5; that is, 64 to 74 inches.

(c) What percent of men are shorter than 66.5 inches? 16%.

(d) A height of 71.5 inches corresponds to what percentile of adult male American heights?
       84%. The area to the left of X = 71.5 under the N(69, 2.5) curve is 0.84.

2.8 **IQ SCORES**   Scores on the Wechsler Adult Intelligence Scale (WAIS, a standard "IQ test") for the 20 to 34 age groups are approximately normally distributed with $\mu = 110$ and $\sigma = 25$. Use the 68–95–99.7 rule to answer these questions.

(a) About what percent of people in this age group have scores above 110? 50%.

(b) About what percent have scores above 160? 2.5%.

(c) In what range do the middle 95% of all IQ scores lie? $m \pm 2s = 110 \pm 50$, or 60 to 160.

2.12 **HELMET SIZES**   The army reports that the distribution of head circumferences among male soldiers is approximately normal with mean 22.8 inches and standard deviation 1.1 inches. Use the 68–95–99.7 rule to answer the following questions.

(a) What percent of soldiers have head circumference greater than 23.9 inches? 16%.

(b) A head circumference of 23.9 inches would be what percentile?   84th percentile. The area to the left of $X = 23.9$ under the $N(22.8, 1.1)$ curve is 0.84.

(c) What percent of soldiers have head circumference between 21.7 inches and 23.9 inches? 68%.

2.15 **WEIGHTS OF DISTANCE RUNNERS**   A study of elite distance runners found a mean body weight of 63.1 kilograms, with a standard deviation of 4.8 kg.

(a) Assuming that the distribution of weights is normal, sketch the density curve of the weight distribution with the horizontal axis marked in kilograms.
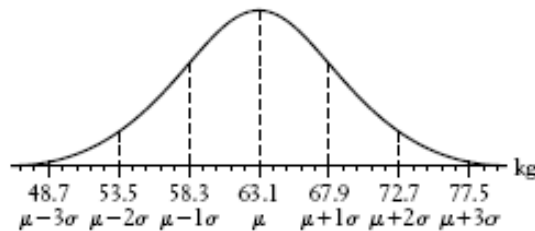
(b) Use the 68-95-99.7 rule to find intervals centered at the mean that will include 68%, 95%, and 99.7% of the weights of the runners.   68%: (58.3, 67.9). 95%: (53.5, 72.7). 99.7%: (48.7, 77.5).

2.16 **CALCULATOR GENERATED DENSITY CURVE**   Like Minitab and similar computer utilities, the TI-84 has a "random number generator" that produces decimal numbers between 0 and 1.
- Press MATH , then choose PRB and 1:Rand.
- Press ENTER several times to see the results, The command 2rand produces a random number between 0 and 2. The density curve of the outcomes has constant height between 0 and 2, and height 0 elsewhere.

(a) What is the height of the density curve between 0 and 2? Draw a graph of the density curve.

(b) Use your graph from (a) and the fact that areas under the curve are relative frequencies of outcomes to find the proportion of outcomes that are less than 1. 50% of the outcomes are less than 1.

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(c) What is the median of the distribution? What are the quartiles? Because the distribution is symmetric, the median is 1, Q1 = 0.5, and Q3 = 1.5.

(d) Find the proportion of outcomes that lie between 0.5 and 1.3. For $0.5 < X < 1.3$, the proportion of outcomes is 0.8 (1/2) = 0.4.

2.19 SAT VERSUS ACT   Eleanor scores 680 on the mathematics part of the SAT. The distribution of SAT scores in a reference population is normal, with mean 500 and standard deviation 100. Gerald takes the American College Testing (ACT) mathematics test and scores 27. ACT scores are normally distributed with mean 18 and standard deviation 6. Find the standardized scores for both students. Assuming that both tests measure the same kind of ability, who has the higher score?

Eleanor's z-score is $\dfrac{680-500}{100} = 1.8$;   Gerald's is $\dfrac{27-18}{6} = 1.5$.
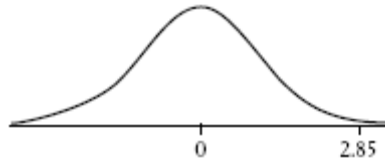
Eleanor's score is higher.

2.21 TABLE A PRACTICE   Use Table A to find the proportion of observations from a standard normal curve that satisfies each of the following statements. In each case, sketch a standard normal curve with your value of z marked on the axis.

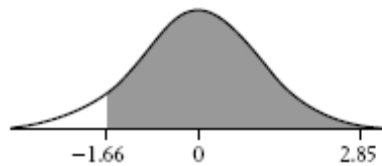(a) $z < 2.85$ 0.9978
[normalcdf(-10, 2.85) = 0.9978]

(b) $z > 2.85$ 1 - 0.9978 = 0.0022
[normalcdf(2.85, 10) = 0.0022]

(c) $z > -1.66$ $1 - 0.0485 = 0.9515$
[normalcdf(-1.66, 10) = 0.9515]

(d) $-1.66 < z < 2.85$ $0.9978 - 0.0485 = 0.9493$
[normalcdf(-1.66, 2.85) = 0.9493]

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2.22 MORE TABLE A PRACTICE Use Table A to find the value z of a standard normal variable that satisfies each of the following conditions. (Use the value of z from Table A that comes closest to satisfying the condition.) In each case, sketch a standard normal curve with your value of z marked on the axis.
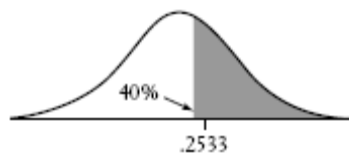
(a) The point z with 25% of the observations falling below it.

25 %

$-.675$
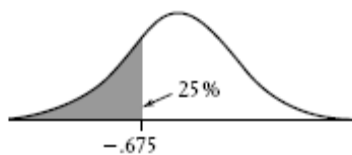
About -0.675 (-0.67449)

invNorm(0.25) = -0.6745]

(b) The point z with 40% of the observations falling above it.


40%

.2533

About 0.25 (0.253347)   [

[invNorm(0.60) = 0.2533]

2.23 HEIGHTS OF AMERICAN MEN The distribution of heights of adult American men is approximately normal with mean 69 inches and standard deviation 2.5 inches.

(a) What percent of men are at least 6 feet (72 inches) tall?

- Let X = height of adult male

- $P(x > 72) = P\left(z > \dfrac{72 - 69}{2.5}\right) = P(z > 1.20) = 0.1151$ [normalcdf(1.20,10)]

- About 11.5% of men are at least 72 inches tall.

Area=.11507
low=1.2   up=10
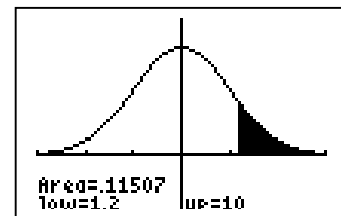
(b) What percent of men are between 5 feet (60 inches) and 6 feet tall?

- Let X = height of adult male

- $P(60 < x < 72) = P\left(\dfrac{60 - 69}{2.5} < z < \dfrac{72 - 69}{2.5}\right) = P(-3.6 < z < 1.2) = 0.8848$ [normalcdf(-3.6,1.2)]

- About 88.5% of men are between 60 inches and 72 inches tall.
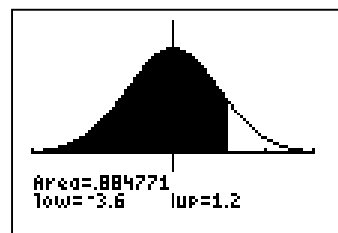
(c) How tall must a man be to be in the tallest 10% of all adult men?

- [invNorm(0.90) = 1.2816]

- Solving $\dfrac{x - 69}{2.5} = 1.2816$ *yields* $x = 72.2$

- A man has to be 72.2 inches to be in the tallest 10% of all adult men.

Area=.884771
low=-3.6   up=1.2

2.24 IQ TEST SCORES Scores on the Wechsler Adult Intelligence Scale (a standard "IQ test") for the 20 to 34 age group are approximately normally distributed with μ = 110 and σ = 25.
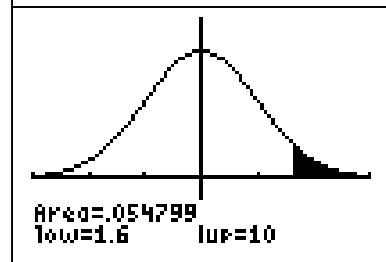
(a) What percent of people aged 20 to 34 have IQ scores above 100?

- X = IQ score of 20 – 34 year old

- $P(x > 100) = P\left(z > \dfrac{100 - 110}{25}\right) = P(z > -0.4) = 0.6554$

- 65.5% of adults age 20 to 34 have IQ scores about 100.
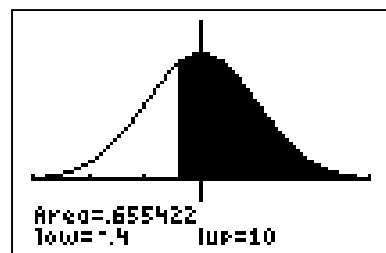
(b) What percent have scores above 150?

- $P(x > 150) = P\left(z > \dfrac{150 - 110}{25}\right) = P(z > 1.6) = 0.0548$

- 5.86% of adults age 20 to 34 have IQ scores about 150.

(c) How high an IQ score is needed to be in the highest 25%?

- [invNorm(0.75) = 0.6745]

- Solving $\dfrac{x - 110}{25} = 0.6745$ *yields* $x = 126.8$

- To be in the highest 25%, an IQ score must be 127 or higher.

Area=.655422
low=-.4   up=10

Area=.054799
low=1.6   up=10

6

2.27 **GREAT WHITE SHARKS** Here are the lengths in feet of 44 great white sharks:

18.7	12.3	18.6	16.4	15.7	18.3	14.6	15.8	14.9	17.6	12.1
16.4	16.7	17.8	16.2	12.6	17.8	13.8	12.2	15.2	14.7	12.4
13.2	15.8	14.3	16.6	9.4	18.2	13.2	13.6	15.3	16.1	13.5
19.1	16.2	22.8	16.8	13.6	13.2	15.7	19.7	18.7	13.2	16.8

(a) Construct a stemplot and describe this distribution of lengths using SOCS.

```
 9 | 4
10 |
11 |
12 | 12346
13 | 22225668
14 | 3679
15 | 237788
16 | 122446788
17 | 688
18 | 23677
19 | 17
20 |
21 |
22 | 8
```

The distribution is approximately symmetric with a peak at the 16 stem.
Median = 15.75
IQR = 3.65
Outliers exist at 9.4 and 22.8.

16|2 means 16.2 feet long

(b) Compare the mean with the median. Does this comparison support your assessment of the shape of the distribution in (a)? $\bar{x}$ = 15.586, Median = 15.75. The similarity of these measures reflects the symmetry observed in part (a)

(d) Use your calculator to construct a normal probability plot. Sketch the plot below and interpret the plot.


P 1:L2
X=12.1   Y=-1.823803
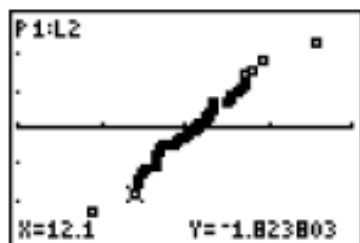
(e) Do you think these data are approximately normal? Write a brief summary of your findings.
The shape of the distribution is similar to that of a normal distribution. Also, the strong linearity of the normal probability plot suggests that the data were drawn from a normal distribution.

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2.31 GESTATION PERIOD The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with mean 266 days and standard deviation 16 days.

(a) What percent of pregnancies last less than 240 days (that's about 8 months)?
- Let X = length of pregnancy
- $P(x < 240) = P\left(z < \dfrac{240 - 266}{16}\right) = P(z < -1.625) = 0.052$ [normalcdf(-10,-1.625)
- About 5.2% of pregnancies last less than 240 days.
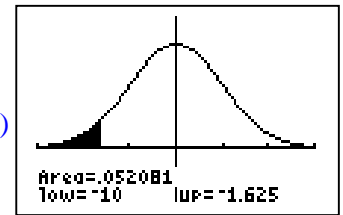
(b) What percent of pregnancies last between 240 and 270 days (roughly between 8 months and 9 months)?
- Let X = length of pregnancy
- $P(240 < x < 270) = P\left(\dfrac{240 - 266}{16} < z < \dfrac{270 - 266}{16}\right) = P(-1.625 < z < 0.25) = 0.5466$
- About 54.7% of pregnancies last less between 240 and 270 days.
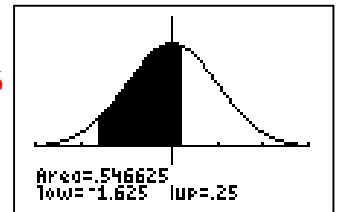
(c) How long do the longest 20% of pregnancies last?
- $invNorm(0.80) = 0.8416$
- $Solving \dfrac{x - 266}{16} = 0.8416, \quad yields\ x = 279.5$
- The longest 20% of pregnancies last at least 280 days.

2.33 QUARTILES The quartiles of any density curve are the points with area 0.25 and 0.75 to their left under the curve.

(a) What are the quartiles of a standard normal distribution? $Q_1 = -0.6745 \quad Q_3 = 0.6745$
$invNorm(0.25) = -0.6745; \quad invNorm(0.75) = 0.6745$

(b) How many standard deviations away from the mean do the quartiles lie an any normal distribution? What are the quartiles for the lengths of human pregnancies? (Use the distribution in 2.31 above). For any normal distribution, the quartiles are ± 0.675 standard deviations from the mean; for human pregnancies, the quartiles are 266 ± 0.675(16) = 266 ± 10.8, or 255.2 and 276.8.
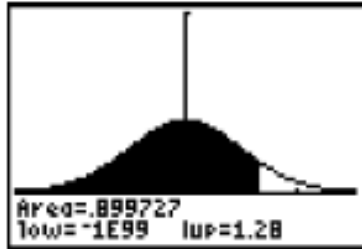
2.40 STANDARDIZED TEST SCORES AS PERCENTILES Joey received a report that he scored in the 97th percentile on a national standardized reading test but in the 72nd percentile on the math portion of the test. Explain to Joey's grandmother, who knows no statistics, what these numbers mean.   Joey's scoring "in the 97th percentile" on the reading test means that Joey scored as well as or better than 97% of all students who took the reading test and scored worse than only 3%.

His scoring in the 72nd percentile on the math portion of the test means that he scored as well as or better than 72% of all students who took the math test and worse than 28%. Joey did better on the reading test, relative to his peers, than he did on the math test.
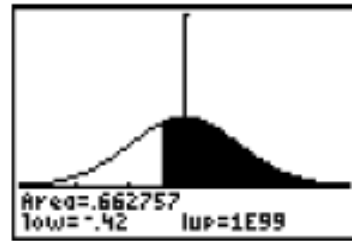
2.41 **TABLE A PRACTICE** Use Table A to find the proportion of observations from a standard normal distribution that falls in each of the following regions. In each case, sketch a standard normal curve and shade that area representing the region.
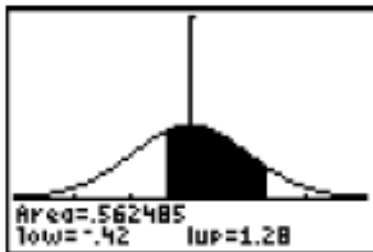
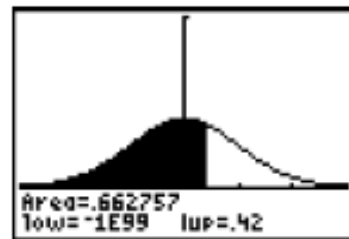(a) $z < 1.28$ 0.8997
[normalcdf(-10, 1.28) = 0.8997]

Area=.899727
low=-1E99   lup=1.28

(b) $z > -0.42$ 0.6628
[normalcdf(-0.42, 10) = 0.6628]

Area=.662757
low=-.42   lup=1E99

(c) $-0.42 < z < 1.28$ 0.5625
[normalcdf(-0.42, 1.28) = 0.5625]

Area=.562485
low=-.42   lup=1.28

(d) $z < 0.42$ 0.6628
[normalcdf(-10, 0.42) = 0.6628]

Area=.662757
low=-1E99   lup=.42
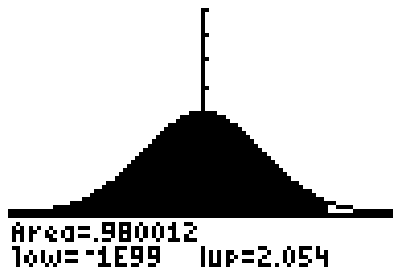
2.42 **WORKING BACKWARDS, FINDING z-VALUES**

(a) Find the number z such that the proportion of observations that are less than z in a standard normal distribution is 0.98. *(sketch the curve)*
   98th percentile of Z = 2.054.
     [invNorm(0.98) = 2.054]

(b) Find the number z such that 22% of all observations from a standard normal distribution are greater than z. *(sketch the curve)*   78th percentile of Z = 0.7722.
     [invNorm(0.78) = 0.7722]

Area=.980012
low=-1E99   lup=2.054

Area=.219998
low=.7722   lup=1E99
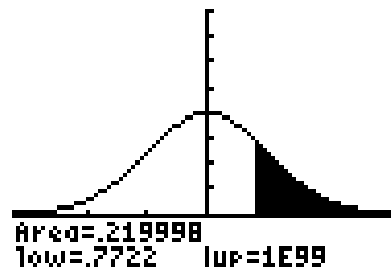
2.45 **IQ SCORES FOR CHILDREN** The scores of a reference population on the Wechsler Intelligence Scale for Children (WISC) are normally distributed with $\mu = 100$ and $\sigma = 15$. A school district classifies children as "gifted" if their WISC scores exceeds 135. There are 1300 sixth-graders in the school district. About how many of them are gifted?

- Let X = score on WISC

- $P(x > 135) = P\left(z > \dfrac{135 - 100}{15}\right) = P(z > 2.33333) = 0.01$

- About 1% of scores are higher than 135. 1% of 1300 = 13. About 13 of the students are gifted.

Area=.009815
low=2.33333 up=10
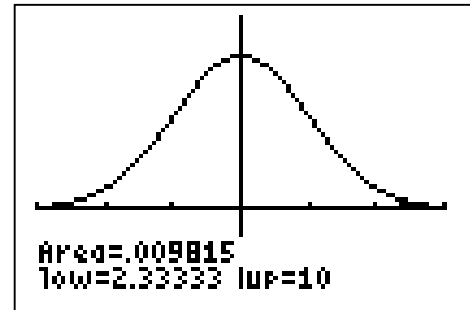
2.47 **HELMET SIZES** The army reports that the distribution of head circumference among soldiers is approximately normal with mean 22.8 inches and standard deviation 1.1 inches. Helmets are mass-produced for all except the smallest 5% and the largest 5% of all head sizes. Soldiers in the smallest or largest 5% get custom-made helmets. What head sizes get custom-made helmets?

invNorm(0.05) = -1.81   invNorm(0.95) = 1.81

Soldiers whose head circumference is outside the range 22.8 ± 1.81(1.1), approximately, less than 20.8 inches or greater than 24.8 inches.